Toulouse School of Economics

M1 INTERMEDIATE ECONOMETRICS

Simultaneity

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Interest lies in (log-)linear demand d and suuply s curves in price p,

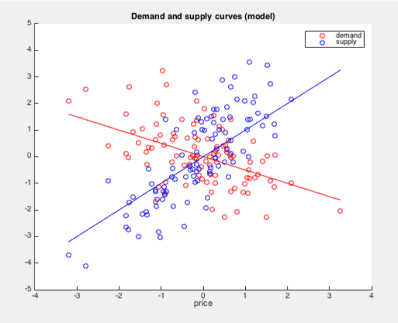
$$d = \alpha_d - \theta_d \, p + u,$$
$$s = \alpha_s \, + \theta_s \, p + v,$$

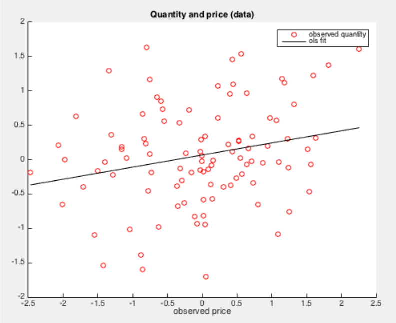
where u, v are unobserved errors.

Wil suppose that the errors are independent and homoskedastic, with $\sigma_u^2 = \mathbb{E}(u^2)$ and $\sigma_v^2 = \mathbb{E}(v^2)$.

Suppose we would like to learn the demand curve; the price elasticity of demand $-\theta_d$ in particular.

Experimental vs observational data





Observational data comes from markets in equilibrium.

Market clearing states that p is determined such that $s = d$ holds.

Plug-in demand and supply curve in equilibrium condition to get

$$\alpha_d - \theta_d\, p + u = \alpha_s + \theta_s\, p + v$$

and solve for the equilibrium price to find

$$p = \frac{\alpha_d - \alpha_s}{\theta_d + \theta_s} + \frac{u - v}{\theta_d + \theta_s}.$$

This also gives traded quantity as

$$q = \frac{\alpha_d \theta_s + \alpha_s \theta_d}{\theta_d + \theta_s} + \frac{\theta_s u + \theta_d v}{\theta_d + \theta_s}.$$

The slope on p in a population regression of q on a constant and p equals

$$\frac{\operatorname{cov}(p,q)}{\operatorname{var}(p)} = \frac{\operatorname{cov}(u-v,\theta_s u + \theta_d v)}{\operatorname{var}(u-v)} = \frac{\sigma_u^2}{\sigma_u^2+\sigma_v^2}\,\theta_s - \frac{\sigma_v^2}{\sigma_u^2+\sigma_v^2}\,\theta_d.$$

This is a weighted average of both the supply and demand elasticities.

So least-squares does not recover any elasticity here.

Endogeneity

We can write

$$d = \alpha_d - \theta_d p + u, \qquad p = \frac{\alpha_d - \alpha_s}{\theta_d + \theta_s} + \frac{u - v}{\theta_d + \theta_s}.$$

Here,

$$\mathbb{E}(pu) = \mathbb{E}\left(u\left(\frac{u - v}{\theta_d + \theta_s}\right)\right) = \frac{\sigma_u^2}{\theta_d + \theta_s} \neq 0,$$

so price is not exogenous but endogenous.

Instrumental variable: Supply shifter

Extend to model by introducing z.

Now

$$\begin{aligned} d &= \alpha_d - \theta_d \, p + u \\ s &= \alpha_s + \theta_s \, p + \gamma z + v \end{aligned}.$$

where $\mathbb{E}(zu) = 0$.

Here, z shifts supply (relevance) but not demand (exclusion).

Now,

$$\begin{aligned} d &= \alpha_d - \theta_d \, p + u \\ p &= \frac{\alpha_d - \alpha_s}{\theta_d + \theta_s} - \frac{\gamma}{\theta_d + \theta_s} \, z + \frac{u - v}{\theta_d + \theta_s} \end{aligned}.$$

Variable z shifts price, thereby creating exogenous variation in p.

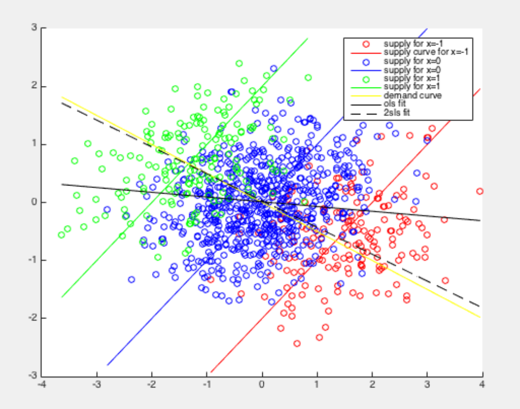
Further, as $\text{cov}(u, z) = 0$,

$$\text{cov}(d, z) = \text{cov}(\alpha_d - \theta_d \, p_i + u, z) = -\theta_d \, \text{cov}(p, z),$$

and so, provided that $\text{cov}(p, z) \neq 0$,

$$-\theta_d = \frac{\text{cov}(d, z)}{\text{cov}(p, z)}$$

can be recovered.


supply for x=-1
supply curve for x=-1
supply for x=0
supply for x=0
supply for x=1
supply for x=1
demand curve
ols fit
2sls fit
3
2
1
0
-1
-2
-3
-4
-3
-2
-1
0
1
2
3
4